Op-Amp Circuits: Part 1

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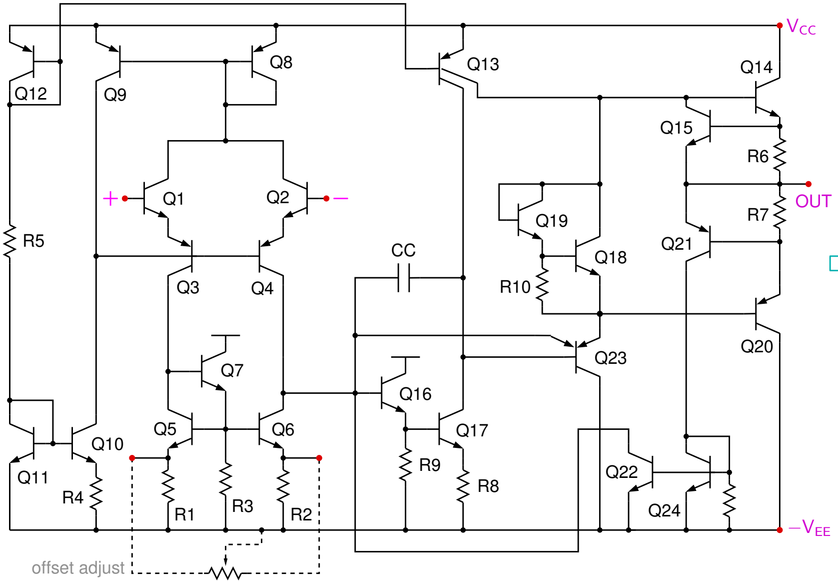
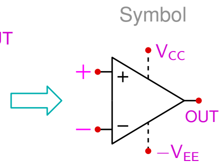
* Amplifiers built with op-amps work with DC input voltages as well $\rightarrow$ useful in sensor applications (e.g., temperature, pressure)
* The user can generally carry out circuit design without a thorough knowledge of the intricate details of an op-amp. This makes the design process simple.

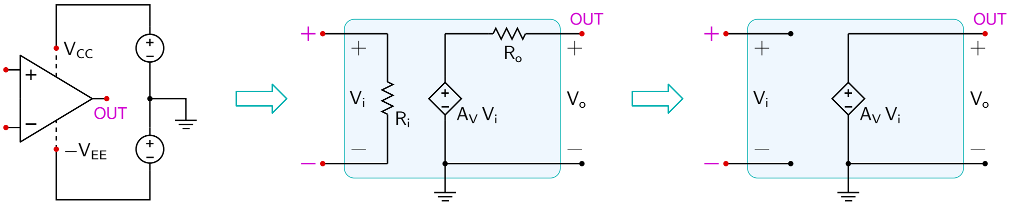


Vcc
Q8
Q13
Q14
Q12
Q9
Q15
R6
Q1
Q2
OUT
R7
Q19
R5
Q18
Q21
CC
Q3
Q4
R10
Q20
Q7
Q23
Q16
Q5
Q6
Q17
Q10
R9
Q22
Q11
R8
R4
R1
R3
R2
Q24
−VEE
offset adjust



Symbol
VCC
OUT
−VEE

Op-amp: equivalent circuit



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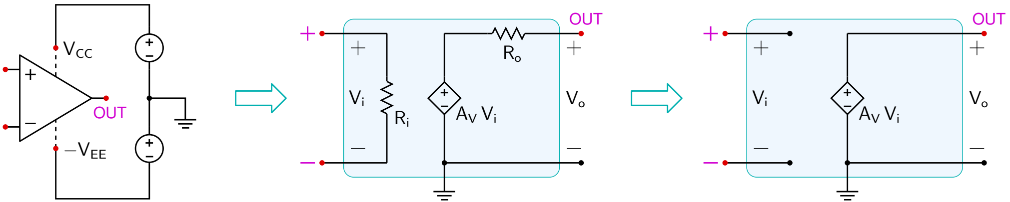

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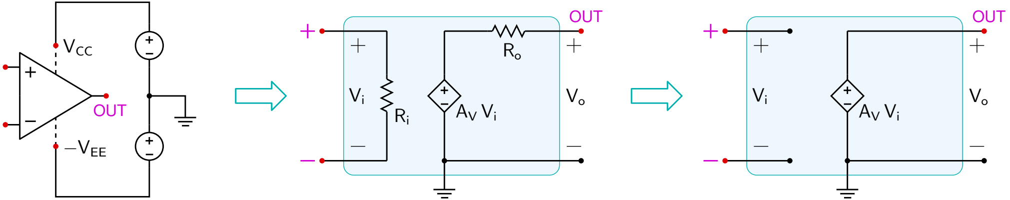

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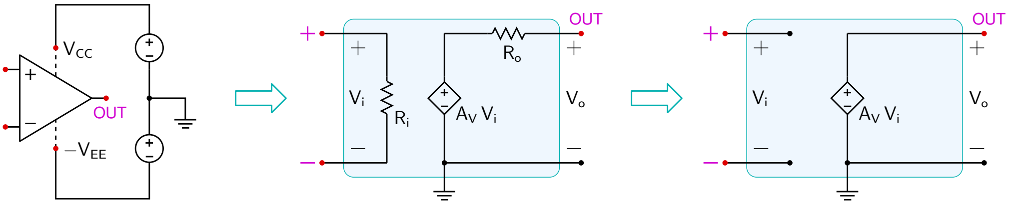

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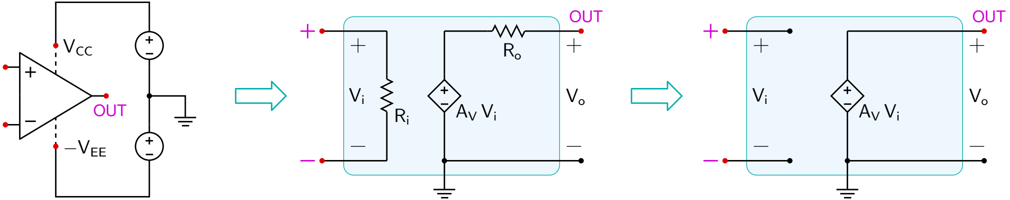

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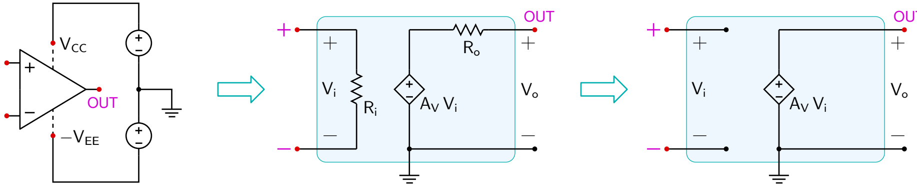
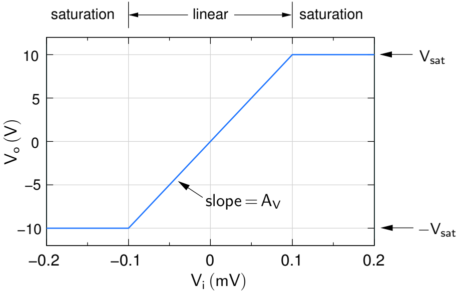
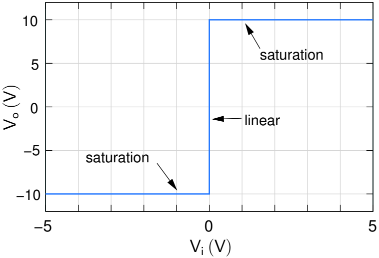
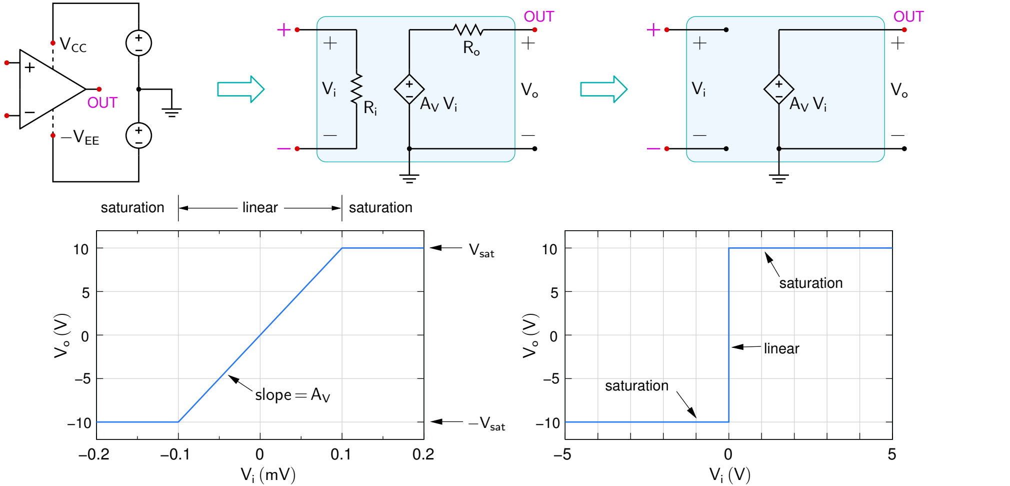
Parameter	Ideal Op-Amp	741
A_V	∞	10^5 (100 dB)
R_i	∞	2 MΩ
R_o	0	75 Ω

Op-Amp: equivalent circuit

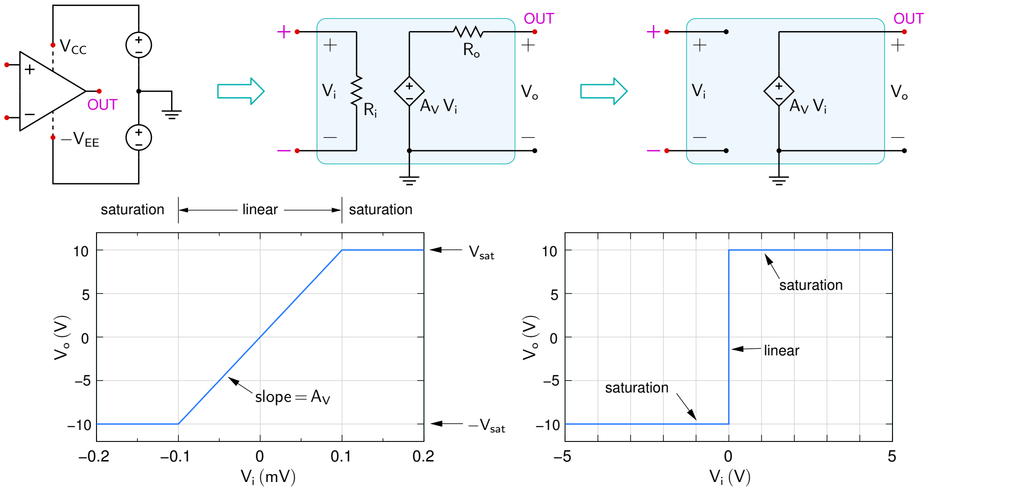




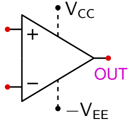




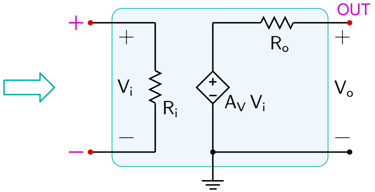
* The output voltage V_o is limited to $\pm V_{\text{sat}}$, where $V_{\text{sat}} \sim 1.5\ V$ less than V_{CC}.



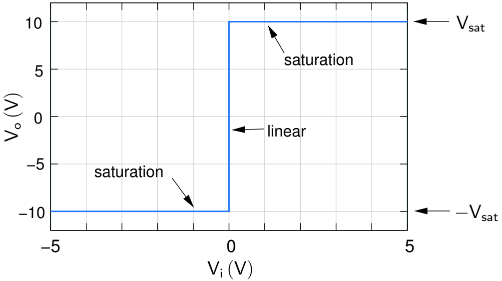
* The output voltage V_o is limited to $\pm V_{\text{sat}}$, where $V_{\text{sat}} \sim 1.5\ V$ less than V_{CC}.
* For $-V_{\text{sat}} < V_o < V_{\text{sat}}$, $V_i = V_+ - V_- = V_o/A_V$, which is very small
  $\rightarrow$ V_+ and V_- are *virtually* the same.



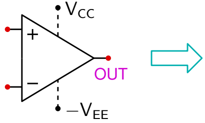
V_{CC}
+
−
OUT
$-V_{EE}$

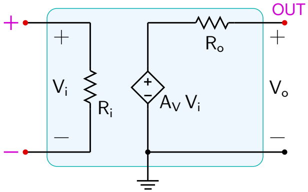


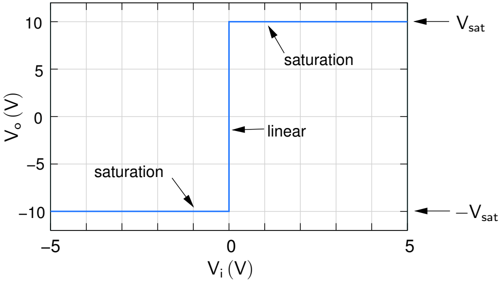
+
−
OUT
R_o
V_i
R_i
$A_V V_i$
V_o



10
5
0
−5
−10
−5
0
5
V_o (V)
V_i (V)
V_{sat}
$-V_{sat}$
saturation
linear
saturation







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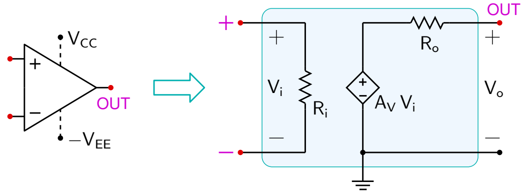

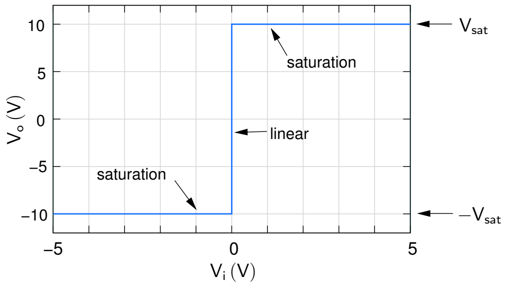

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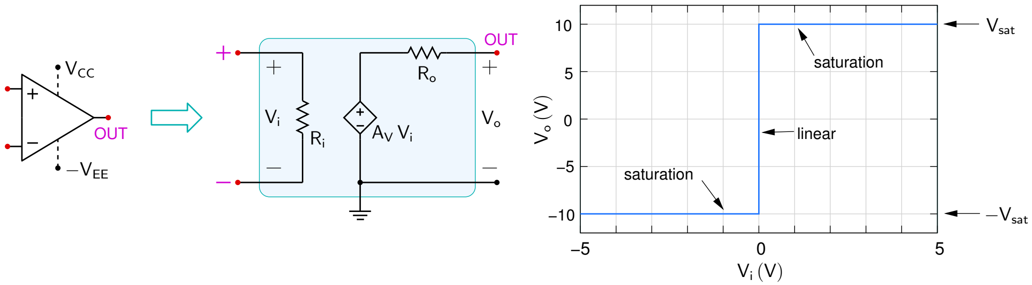

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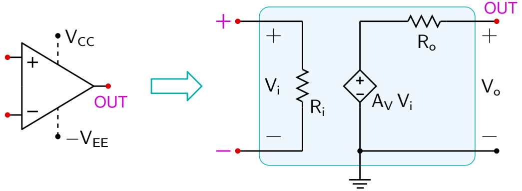
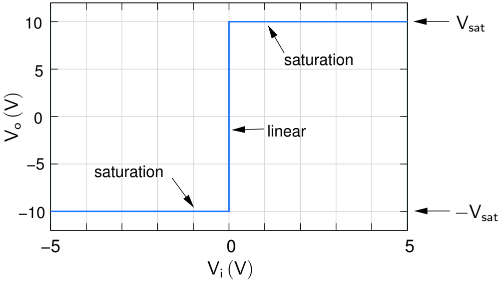
  - op-amp operating in the linear region
  - op-amp operating in the saturation region





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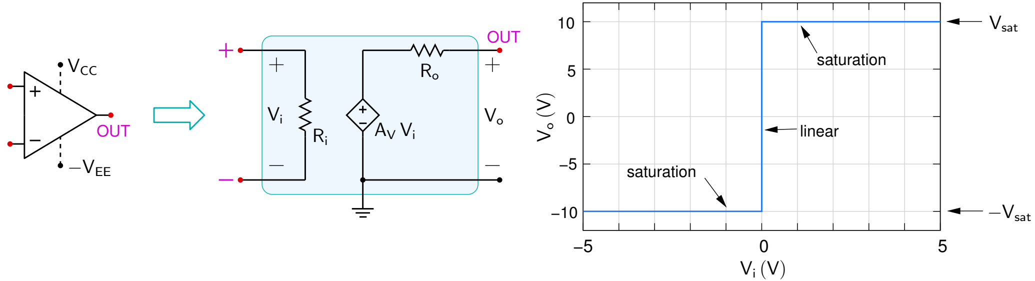
Op-amp circuits



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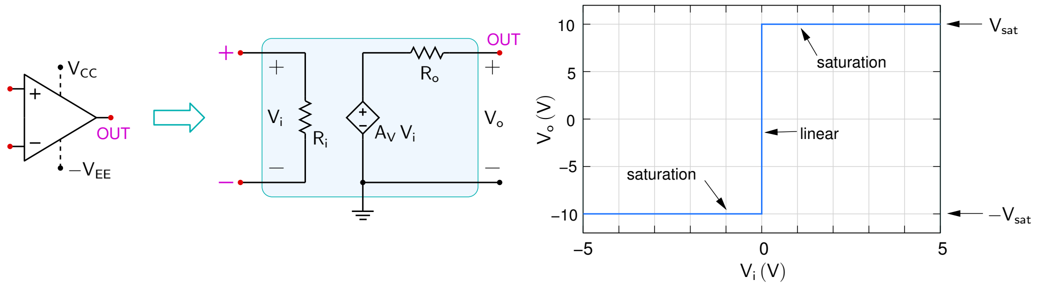

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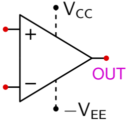
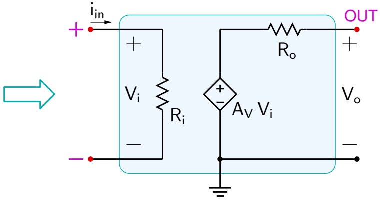
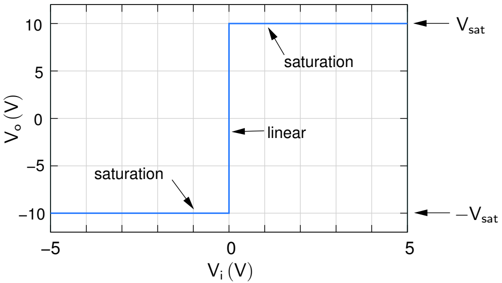
  - type of feedback (negative or positive)
    (We will take a qualitative look at feedback later.)

Op-amp circuits (linear region)







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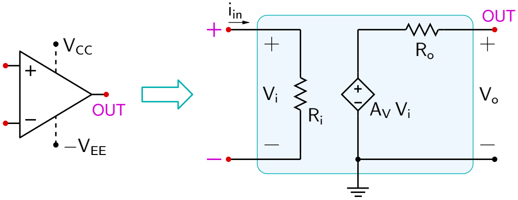

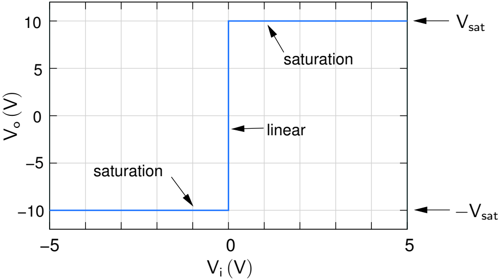

In the linear region,

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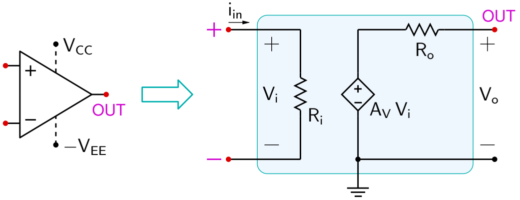

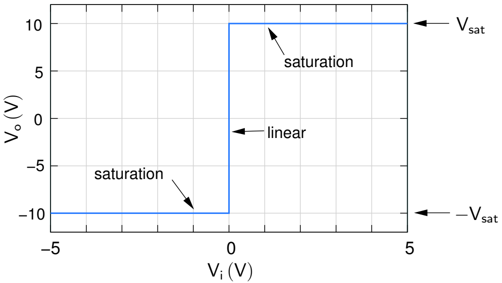

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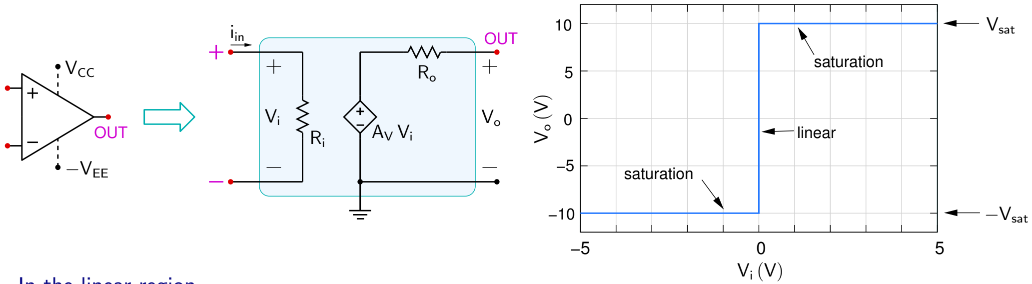

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These two "golden rules" enable us to understand several op-amp circuits.

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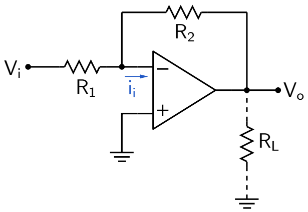

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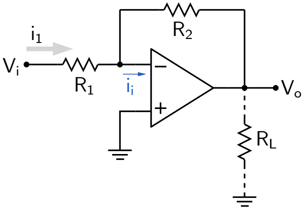

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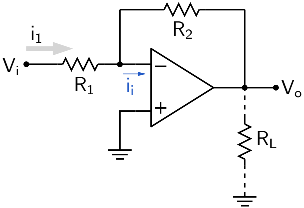

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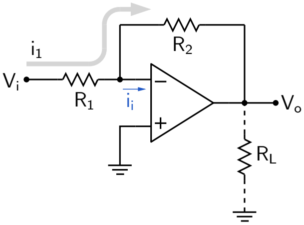

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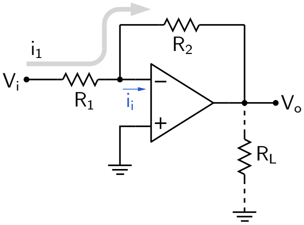

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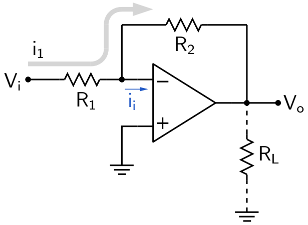

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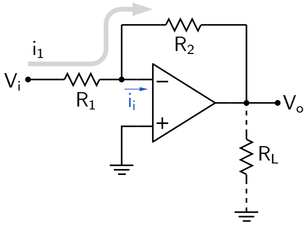

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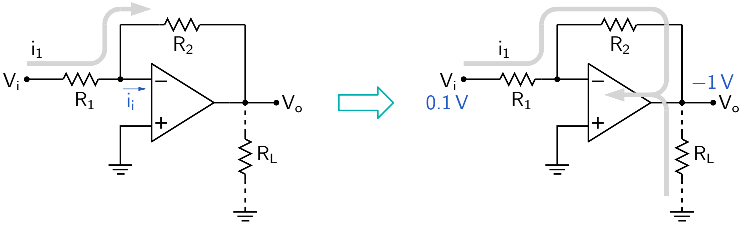
Where does the current go?



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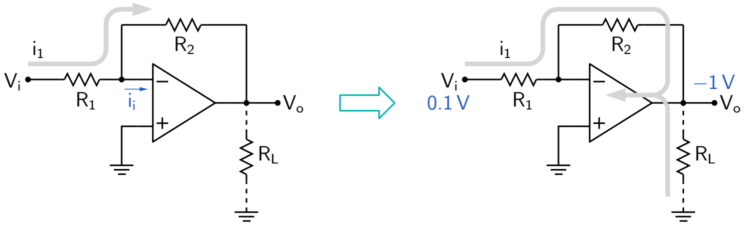

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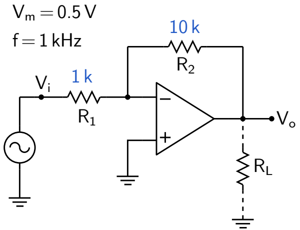
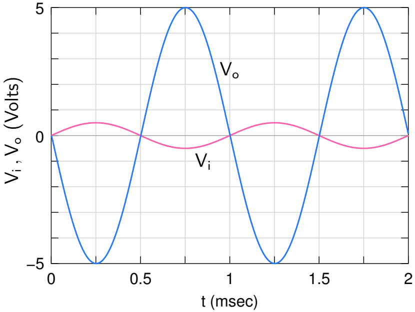
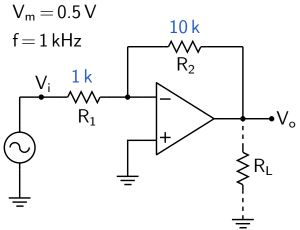
Where does the current go?

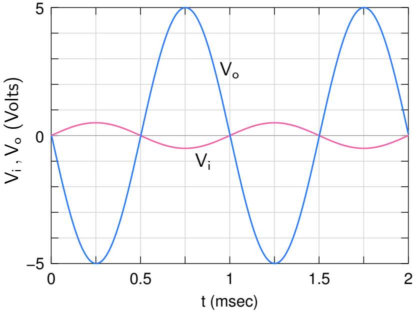
(Op-amp 741 can source or sink about 25 mA.)

Op-amp circuits: inverting amplifier

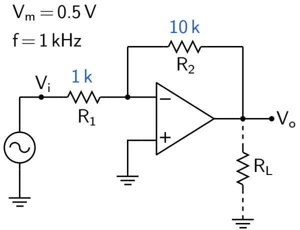


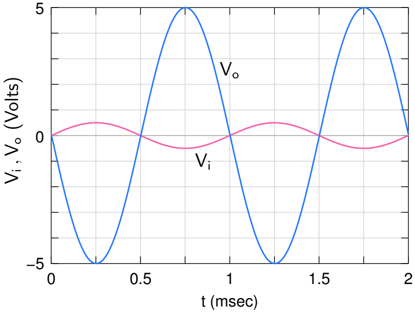






* The gain of the inverting amplifier is $-R_2/R_1$. It is called the "closed-loop gain" (to distinguish it from the "open-loop gain" of the op-amp which is $\sim 10^5$).





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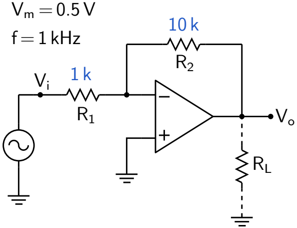

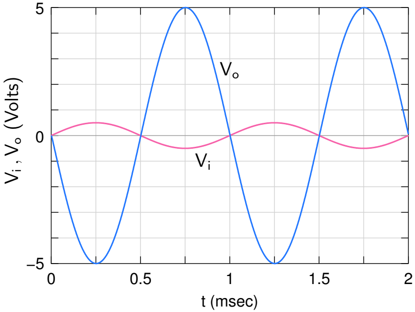

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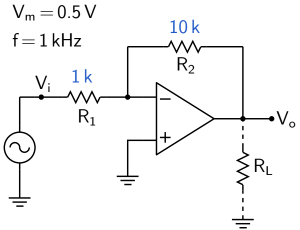
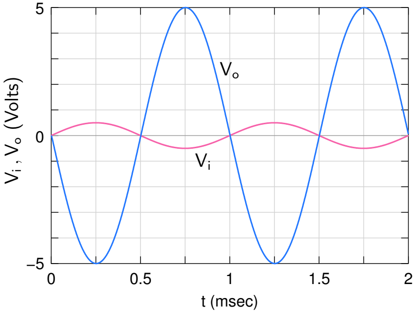
* For the common-emitter amplifier, on the other hand, the gain $-g_m\,(R_C \parallel R_L)$ depends on how the BJT is biased (since g_m depends on I_C).

Op-amp circuits: inverting amplifier





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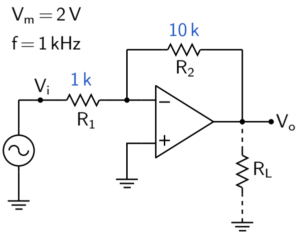
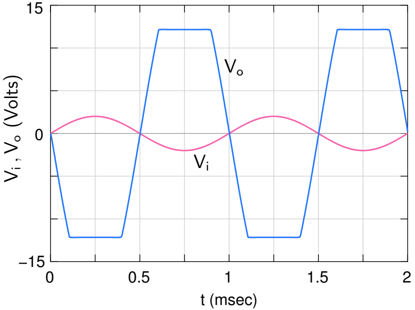
* The gain can be adjusted simply by changing R_1 or R_2 !
* For the common-emitter amplifier, on the other hand, the gain $-g_m\,(R_C \parallel R_L)$ depends on how the BJT is biased (since g_m depends on I_C).

(SEQUEL file: `ee101_inv_amp_1.sqproj`)

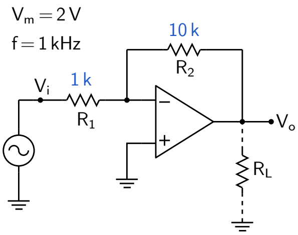


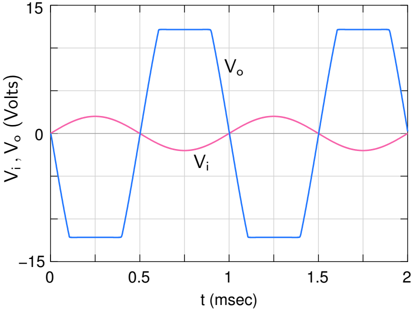
$V_m = 2\,V$
$f = 1\,kHz$
10 k
R_2
V_i
1 k
R_1
V_o
R_L



15
0
−15
V_i , V_o (Volts)
V_o
V_i
0
0.5
1
1.5
2
t (msec)

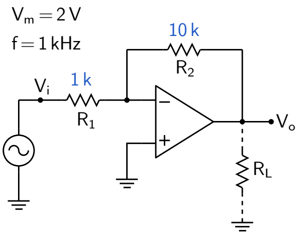
Op-amp circuits: inverting amplifier

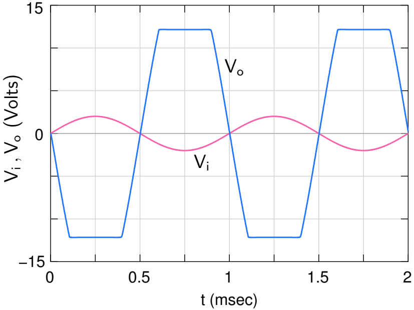




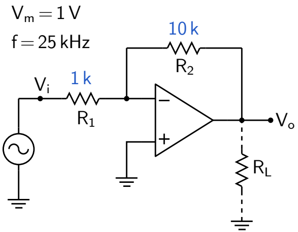
* The output voltage is limited to $\pm V_{sat}$.

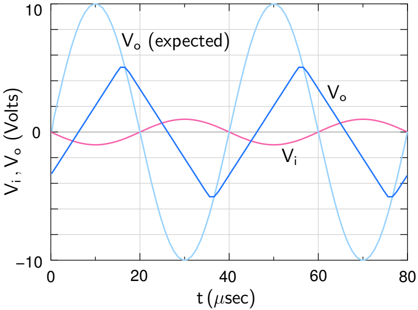
Op-amp circuits: inverting amplifier



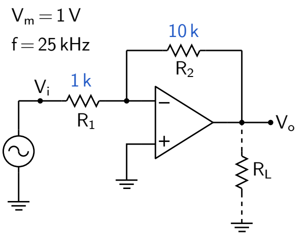


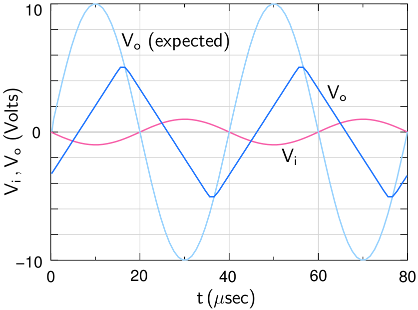
* The output voltage is limited to $\pm V_{sat}$.
* V_{sat} is ~ 1.5 V less than the supply voltage V_{CC}.


$V_m = 1\,V$
$f = 25\,kHz$
10 k
R_2
V_i
1 k
R_1
V_o
R_L

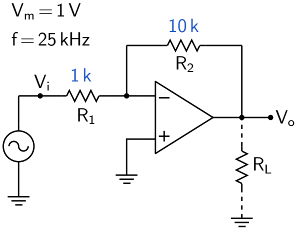

V_o (expected)
V_o
V_i
V_i, V_o (Volts)
10
0
−10
0
20
40
60
80
t (μsec)

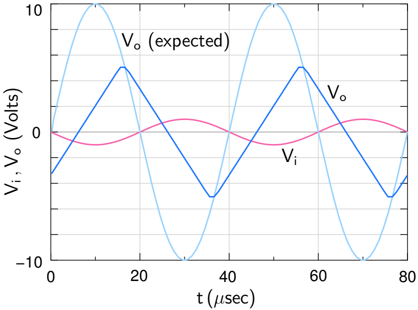
Op-amp circuits: inverting amplifier



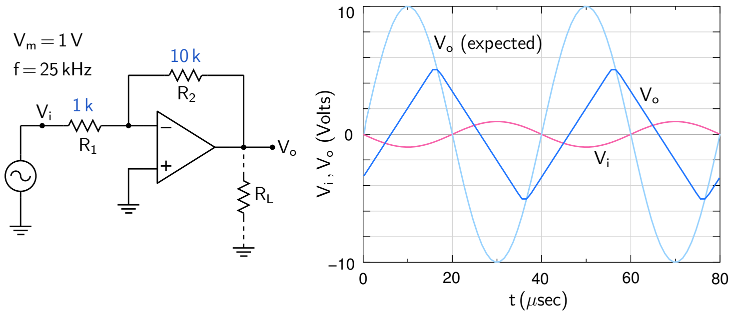


∗ If the signal frequency is too high, a practical op-amp cannot keep up with the input due to its "slew rate" limitation.





* If the signal frequency is too high, a practical op-amp cannot keep up with the input due to its "slew rate" limitation.
* The slew rate of an op-amp is the maximum rate at which the op-amp output can rise (or fall).



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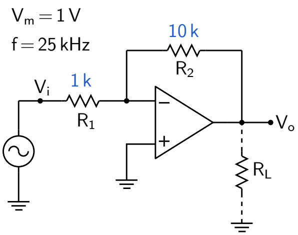
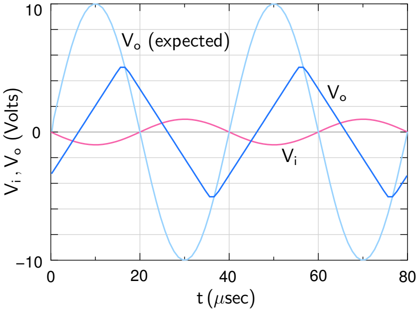
* For the 741, the slew rate is 0.5 V/μsec.

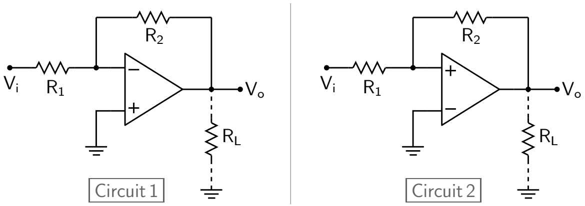




* If the signal frequency is too high, a practical op-amp cannot keep up with the input due to its "slew rate" limitation.
* The slew rate of an op-amp is the maximum rate at which the op-amp output can rise (or fall).
* For the 741, the slew rate is 0.5 V/μsec.

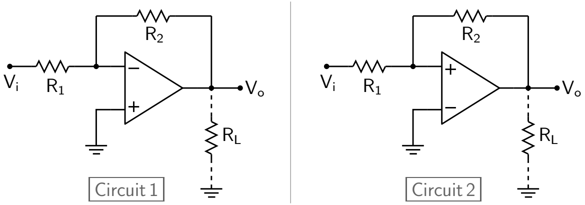
(SEQUEL file: `ee101_inv_amp_2.sqproj`)

Op-amp circuits: inverting amplifier



What if the + (non-inverting) and − (inverting) inputs of the op-amp are interchanged?

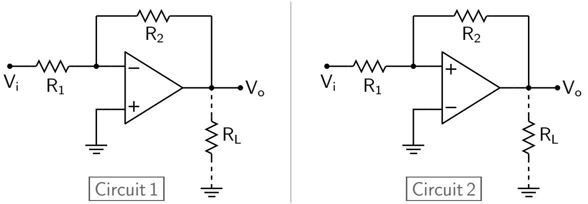
Op-amp circuits: inverting amplifier



What if the + (non-inverting) and − (inverting) inputs of the op-amp are interchanged?

Our previous analysis would once again give us $V_o = -\dfrac{R_2}{R_1}\, V_i$.

Op-amp circuits: inverting amplifier

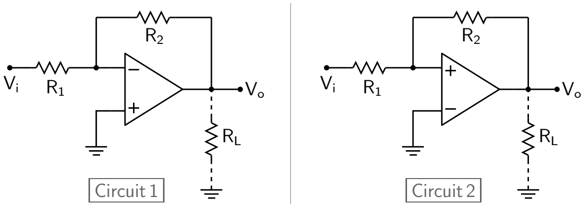


What if the $+$ (non-inverting) and $-$ (inverting) inputs of the op-amp are interchanged?

Our previous analysis would once again give us $V_o = -\dfrac{R_2}{R_1}\, V_i$.

However, from Circuit 1 to Circuit 2, the nature of the feedback changes from negative to positive.

$\rightarrow$ Our assumption that the op-amp is working in the linear region does not hold for Circuit 2, and $V_o = -\dfrac{R_2}{R_1}\, V_i$ does not apply any more.



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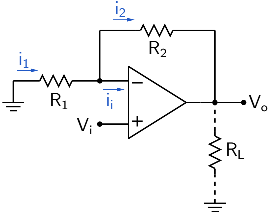
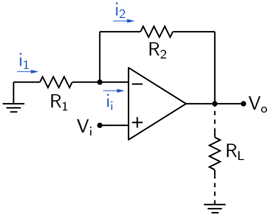
→ Our assumption that the op-amp is working in the linear region does not hold for Circuit 2, and $V_o = -\dfrac{R_2}{R_1}\,V_i$ does not apply any more.

(Circuit 2 is also useful, and we will discuss it later.)

Op-amp circuits (linear region)

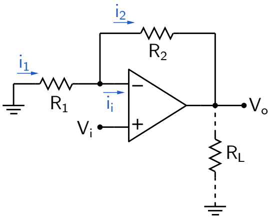


* $V_+ \approx V_- = V_i$

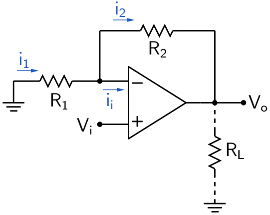


* $V_+ \approx V_- = V_i$

  $\rightarrow i_1 = (0 - V_i)/R_1 = -V_i/R_1$.



* $V_+ \approx V_- = V_i$
  $\rightarrow i_1 = (0 - V_i)/R_1 = -V_i/R_1$.
* Since $i_i = 0$, $i_2 = i_1 \rightarrow V_o = V_- - i_2\, R_2 = V_+ - i_1\, R_2 = V_i - \left(-\dfrac{V_i}{R_1}\right) R_2 = V_i \left(1 + \dfrac{R_2}{R_1}\right)$.



* $V_+ \approx V_- = V_i$
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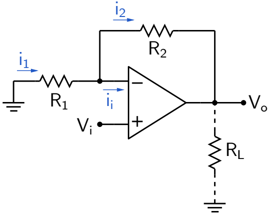
* This circuit is known as the "non-inverting amplifier."



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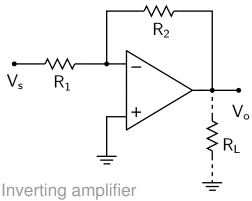
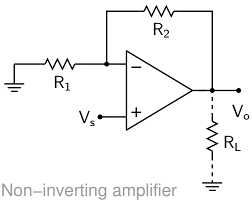
* This circuit is known as the "non-inverting amplifier."
* Again, interchanging + and − changes the nature of the feedback from negative to positive, and the circuit operation becomes completely different.

Inverting or non-inverting?



$$V_o = -\frac{R_2}{R_1} V_s$$

Inverting amplifier

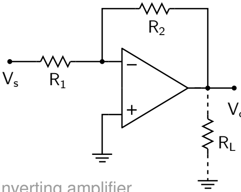


$$V_o = \left(1 + \frac{R_2}{R_1}\right) V_s$$

Non–inverting amplifier

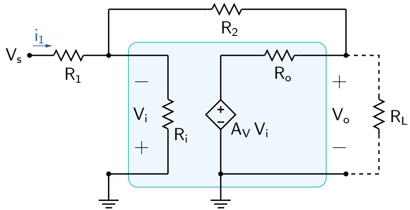
* If the sign of the output voltage is not a concern, which configuration should be preferred?

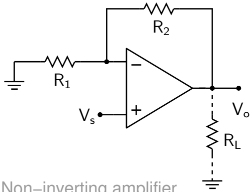
Inverting or non-inverting?



$$V_o = -\frac{R_2}{R_1}\, V_s$$

Inverting amplifier





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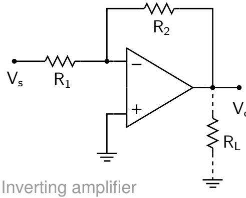
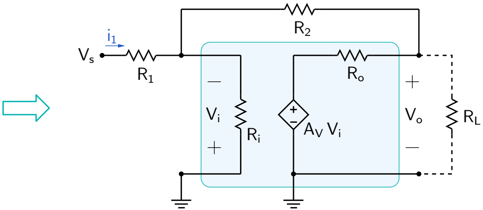
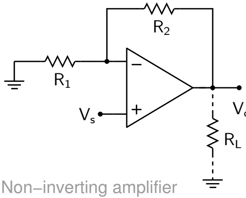


$$V_o = -\frac{R_2}{R_1} V_s$$

Inverting amplifier



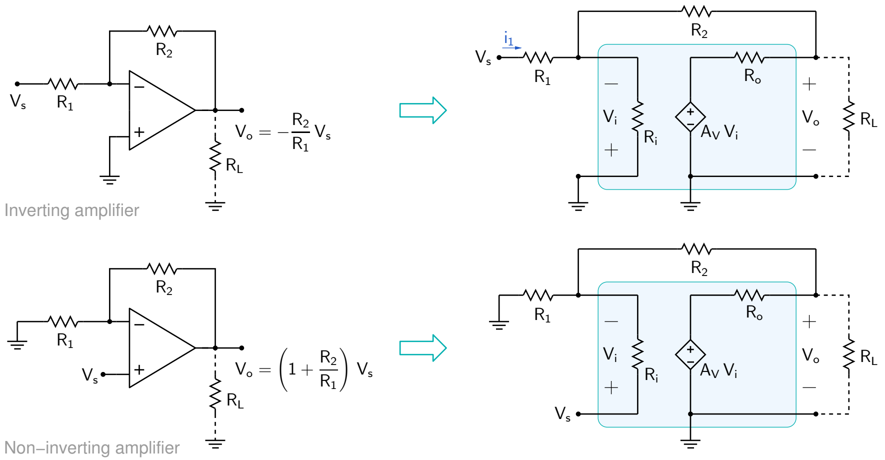


$$V_o = \left(1 + \frac{R_2}{R_1}\right) V_s$$

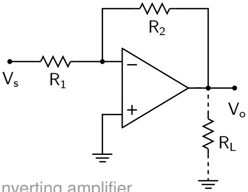
Non–inverting amplifier

* If the sign of the output voltage is not a concern, which configuration should be preferred?
* For the inverting amplifier, since $V_- \approx 0\ V$, $i_1 = V_s/R_1 \rightarrow R_{in} = V_s/i_1 = R_1$.

Inverting or non-inverting?

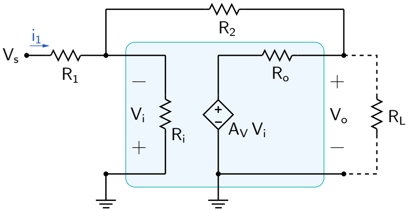


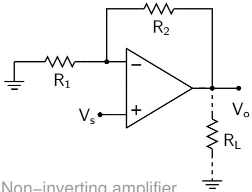
- ∗ If the sign of the output voltage is not a concern, which configuration should be preferred?
- ∗ For the inverting amplifier, since $V_- \approx 0\,V$, $i_1 = V_s/R_1 \rightarrow R_{in} = V_s/i_1 = R_1$.



$$V_o = -\frac{R_2}{R_1}\,V_s$$

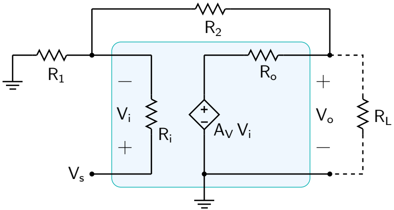
Inverting amplifier





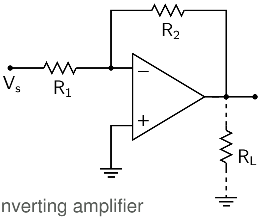
$$V_o = \left(1 + \frac{R_2}{R_1}\right) V_s$$

Non–inverting amplifier



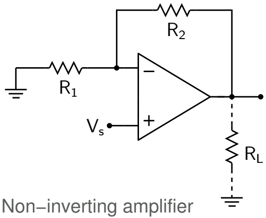
* If the sign of the output voltage is not a concern, which configuration should be preferred?
* For the inverting amplifier, since $V_- \approx 0\ V$, $i_1 = V_s/R_1 \rightarrow R_{\text{in}} = V_s/i_1 = R_1$.
* For the non-inverting amplifier, $R_{\text{in}} \sim R_i\, A_V\, \dfrac{R_1}{R_1 + R_2}$. Huge!

Inverting and non-inverting amplifiers: summary



$$V_o = -\frac{R_2}{R_1}\,V_s$$

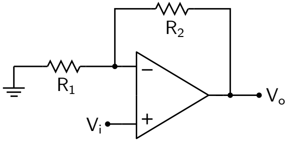
Inverting amplifier



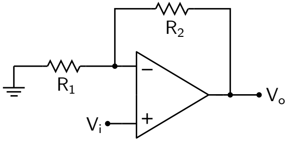
$$V_o = \left(1 + \frac{R_2}{R_1}\right) V_s$$

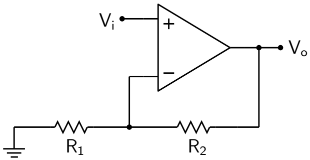
Non–inverting amplifier

Non-inverting amplifier

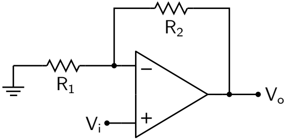


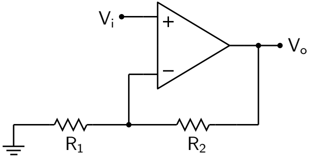
Non-inverting amplifier

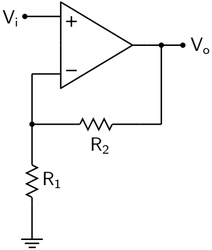




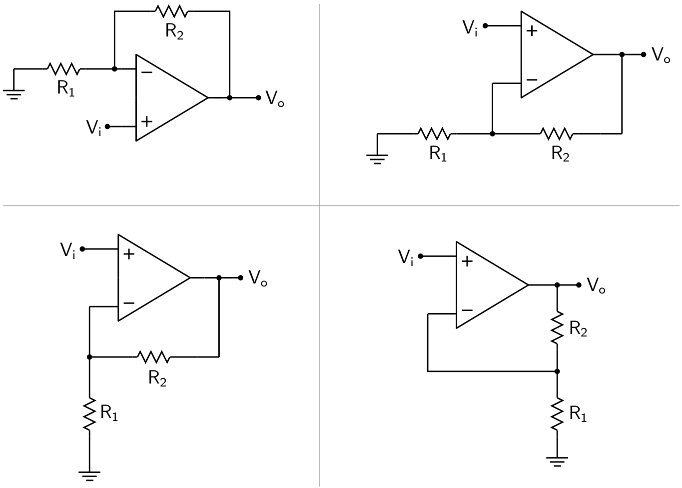
Non-inverting amplifier

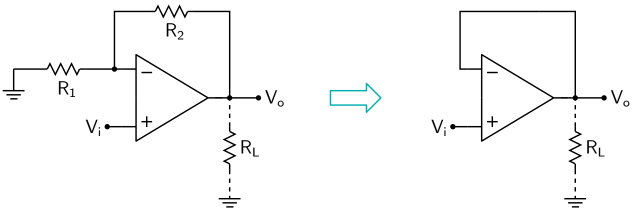






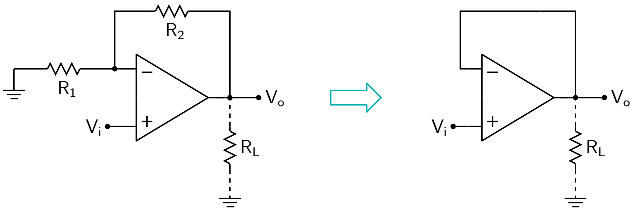
Non-inverting amplifier





Consider $R_1 \to \infty\,,\;\; R_2 \to 0\,.$

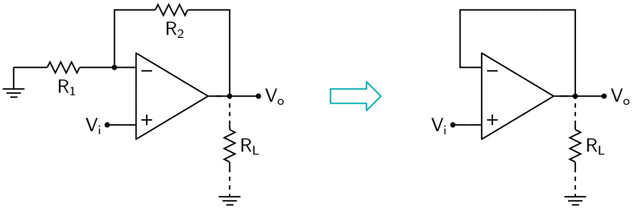
Non-inverting amplifier



Consider $R_1 \rightarrow \infty\,,\;\; R_2 \rightarrow 0\,.$

$$\frac{V_o}{V_i} \rightarrow 1 + \frac{R_2}{R_1} \rightarrow 1\,,\ \text{i.e.,}\ V_o = V_i\,.$$

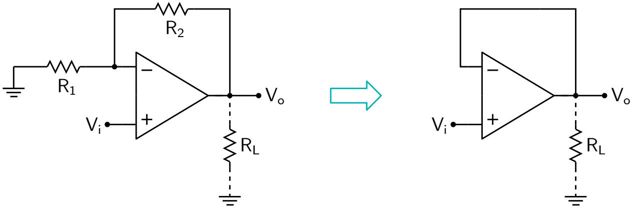
Non-inverting amplifier



Consider $R_1 \to \infty\,,\;\; R_2 \to 0\,.$

$\dfrac{V_o}{V_i} \to 1 + \dfrac{R_2}{R_1} \to 1$, i.e., $V_o = V_i\,.$

This circuit is known as unity-gain amplifier/voltage follower/buffer.



Consider $R_1 \rightarrow \infty\,,\ \ R_2 \rightarrow 0\,.$

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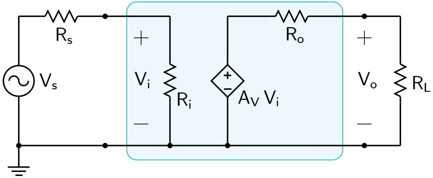
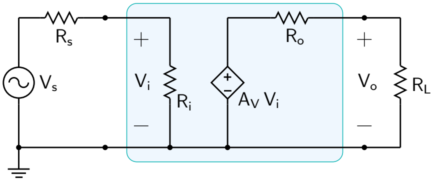
This circuit is known as unity-gain amplifier/voltage follower/buffer.

What has been achieved?



Consider an amplifier of gain A_V. We would like to have $V_o = A_V\, V_s$.

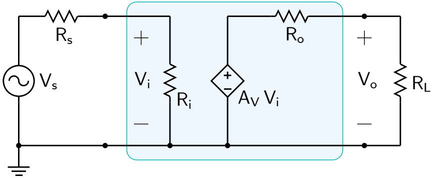


Consider an amplifier of gain A_V. We would like to have $V_o = A_V\, V_s$.

However, the actual output voltage is,

$$V_o = \frac{R_L}{R_o + R_L}\, A_V\, V_i = A_V\, \frac{R_L}{R_o + R_L}\, \frac{R_i}{R_i + R_s}\, V_s\, .$$

Loading effects



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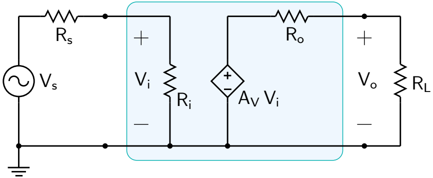
To obtain the desired V_o, we need $R_i \rightarrow \infty$ and $R_o \rightarrow 0$.

Loading effects



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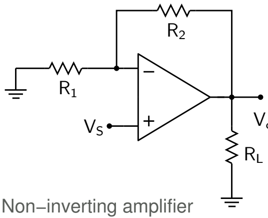
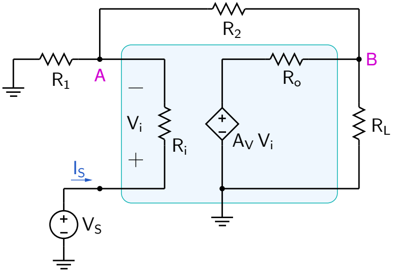
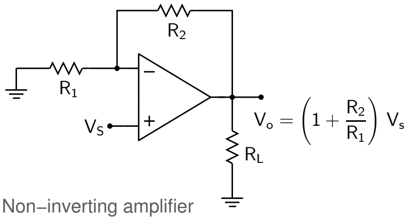
To obtain the desired V_o, we need $R_i \to \infty$ and $R_o \to 0$.

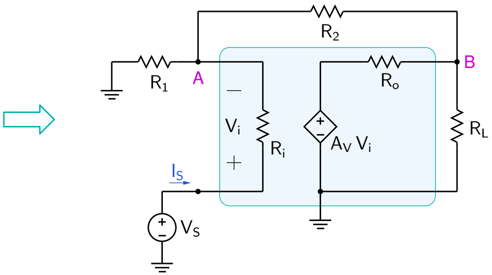
The buffer (voltage follower) provides these features.



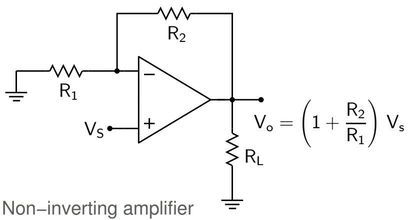
$$V_o = \left(1 + \frac{R_2}{R_1}\right) V_s$$

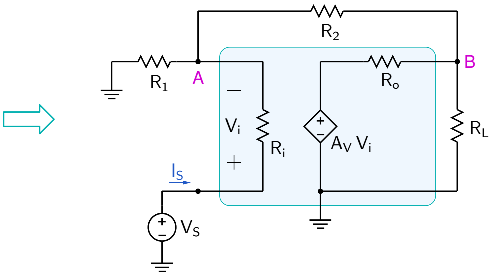






KCL at B: $\dfrac{V_B}{R_L} + \dfrac{V_B - A_V V_i}{R_o} + \dfrac{V_B - V_A}{R_2} = 0.$





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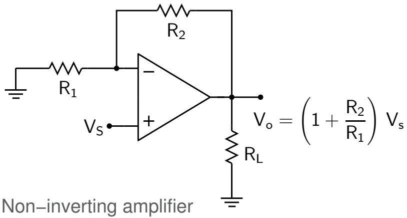
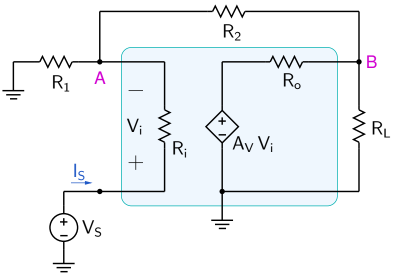
Source current: $$I_S = \frac{V_A}{R_1} + \frac{V_A - V_B}{R_2}.$$

Op-amp buffer: input resistance



Non–inverting amplifier



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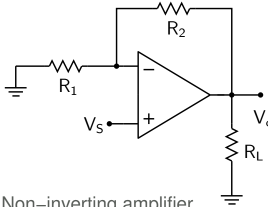
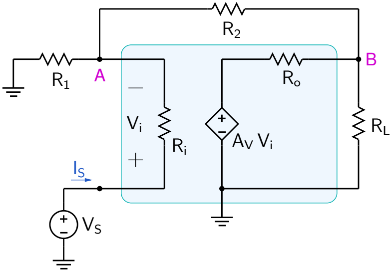
Using $V_i = I_S R_i$, $V_A = V_S - V_i$, and after some algebra, we get

$$R_{\text{in}} = \frac{V_S}{I_S} = \frac{\left(1 + \frac{R_o}{R_L} + \frac{R_o}{R_2}\right) + R_i\left[\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\left(1 + \frac{R_o}{R_L} + \frac{R_o}{R_2}\right) - \frac{R_o}{R_2^2} + \frac{A_V}{R_2}\right]}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\left(1 + \frac{R_o}{R_L} + \frac{R_o}{R_2}\right) - \frac{R_o}{R_2^2}}.$$



Non–inverting amplifier

$$V_o = \left(1 + \frac{R_2}{R_1}\right) V_s$$



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Using $V_i = I_S R_i$, $V_A = V_S - V_i$, and after some algebra, we get

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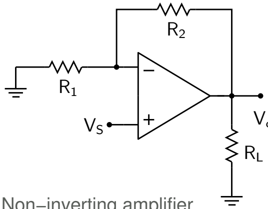
Non-inverting amplifier: input resistance (continued)



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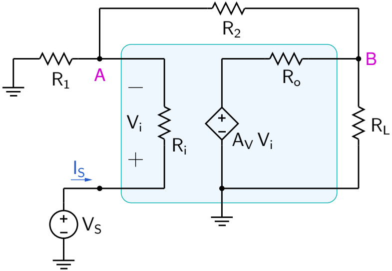
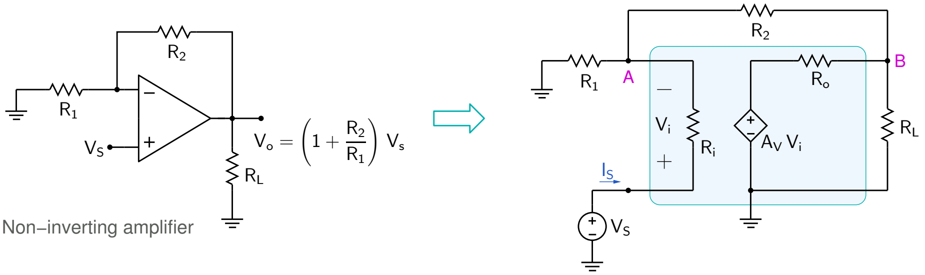
Non–inverting amplifier



$$R_{\text{in}} = \frac{V_S}{I_S} = \frac{\left(1 + \frac{R_o}{R_L} + \frac{R_o}{R_2}\right) + R_i\left[\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\left(1 + \frac{R_o}{R_L} + \frac{R_o}{R_2}\right) - \frac{R_o}{R_2^2} + \frac{A_V}{R_2}\right]}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\left(1 + \frac{R_o}{R_L} + \frac{R_o}{R_2}\right) - \frac{R_o}{R_2^2}}.$$

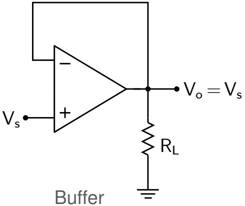


$$R_{\rm in}=\frac{V_S}{I_S}=\frac{\left(1+\dfrac{R_o}{R_L}+\dfrac{R_o}{R_2}\right)+R_i\left[\left(\dfrac{1}{R_1}+\dfrac{1}{R_2}\right)\left(1+\dfrac{R_o}{R_L}+\dfrac{R_o}{R_2}\right)-\dfrac{R_o}{R_2^2}+\dfrac{A_V}{R_2}\right]}{\left(\dfrac{1}{R_1}+\dfrac{1}{R_2}\right)\left(1+\dfrac{R_o}{R_L}+\dfrac{R_o}{R_2}\right)-\dfrac{R_o}{R_2^2}}.$$

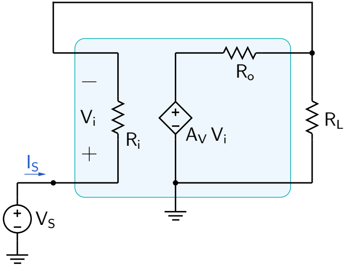
Since R_o is much smaller than R_1, R_2, R_L, or R_i,

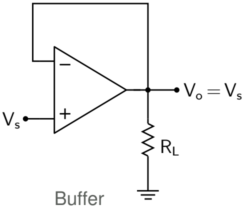
$$R_{\rm in}\approx\frac{1+R_i\left[\left(\dfrac{1}{R_1}+\dfrac{1}{R_2}\right)+\dfrac{A_V}{R_2}\right]}{\left(\dfrac{1}{R_1}+\dfrac{1}{R_2}\right)}\approx\frac{R_i\left[\dfrac{R_1+R_2}{R_1R_2}+\dfrac{A_V}{R_2}\right]}{\dfrac{R_1+R_2}{R_1R_2}}\approx A_VR_i\,\frac{R_1}{R_1+R_2}.$$

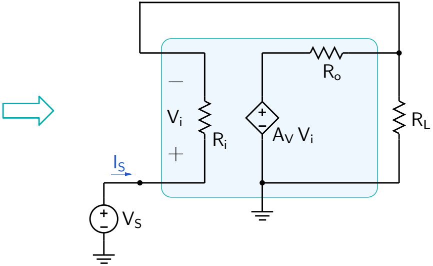
Op-amp buffer: input resistance



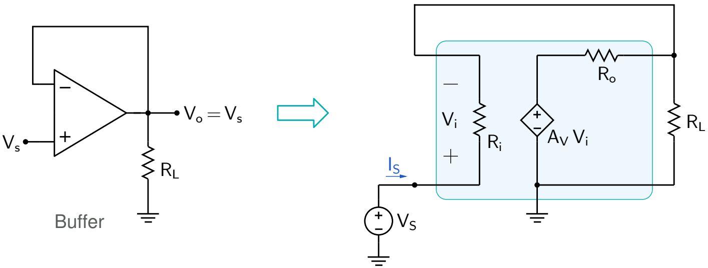
Buffer





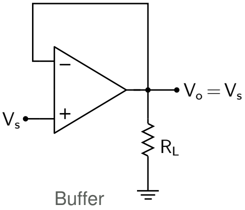


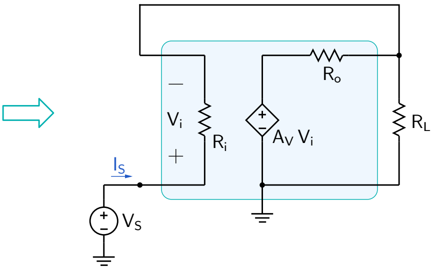
Let $R_o \rightarrow 0$.



Let $R_o \rightarrow 0$.

$V_S = V_i + A_V V_i = V_i(1 + A_V)$.

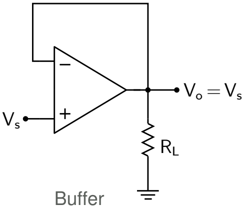


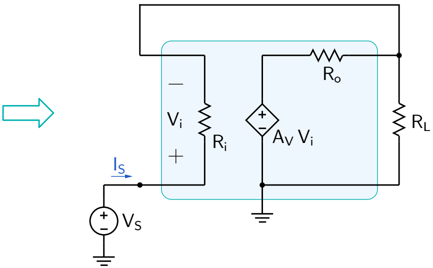


Let $R_o \rightarrow 0$.

$V_S = V_i + A_V V_i = V_i(1 + A_V)$.

$I_S = \dfrac{V_i}{R_i}$.





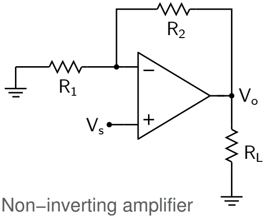
Let $R_o \rightarrow 0$.

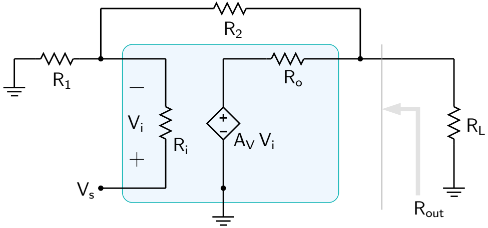
$V_S = V_i + A_V V_i = V_i(1 + A_V)$.

$I_S = \dfrac{V_i}{R_i}$.

$\rightarrow R_{\text{in}} = \dfrac{V_S}{I_S} = R_i(A_V + 1)$

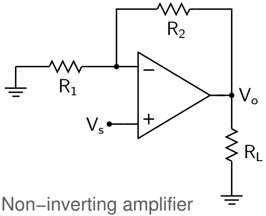
Op-amp buffer: output resistance



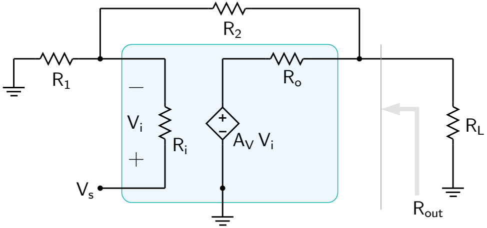


To find R_{out},

* Deactivate the input source.



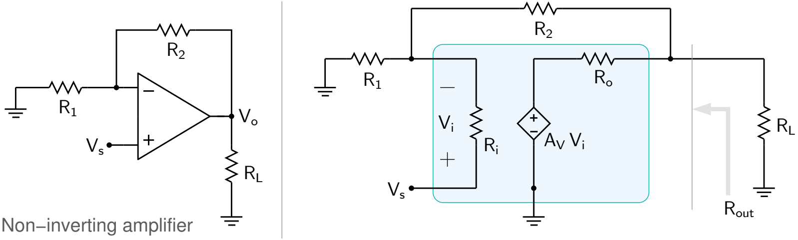
Non–inverting amplifier



To find R_{out},

- ∗ Deactivate the input source.
- ∗ Replace R_L with a test source V'.

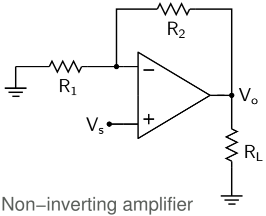
Op-amp buffer: output resistance

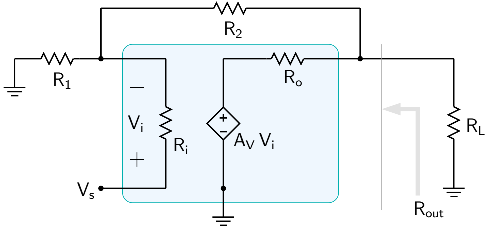


To find R_{out},

* Deactivate the input source.
* Replace R_L with a test source V'.
* Find the current (I') through V'.

Op-amp buffer: output resistance

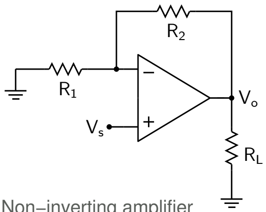




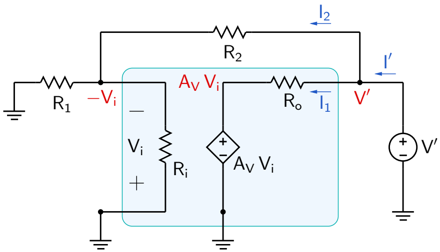
To find R_{out},

- ∗ Deactivate the input source.
- ∗ Replace R_L with a test source V'.
- ∗ Find the current (I') through V'.
- ∗ $R_{\text{out}} = \dfrac{V'}{I'}$.

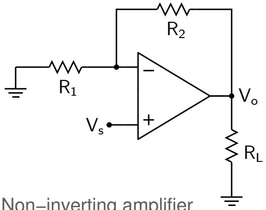
Op-amp buffer: output resistance (continued)



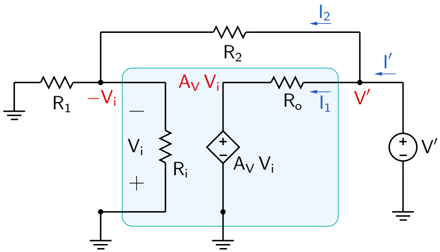
Non–inverting amplifier



Op-amp buffer: output resistance (continued)

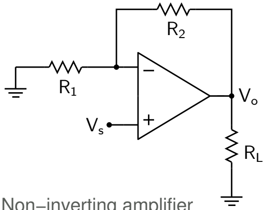


Non–inverting amplifier

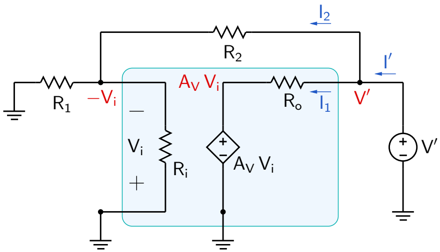


$$V_i = -\frac{(R_i \parallel R_1)}{R_2 + (R_i \parallel R_1)}\, V' \equiv -kV'.$$

Op-amp buffer: output resistance (continued)



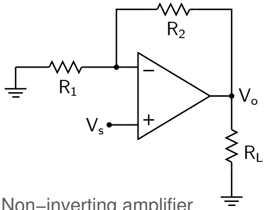
Non–inverting amplifier



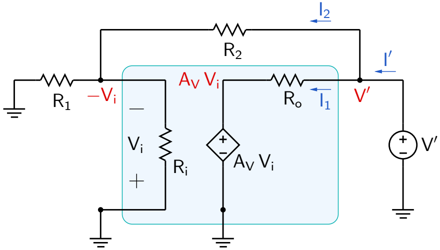
$$V_i = -\frac{(R_i \parallel R_1)}{R_2 + (R_i \parallel R_1)} \, V' \equiv -kV'.$$

$$I' = I_1 + I_2 = \frac{V' - A_V V_i}{R_o} + \frac{V' - (-V_i)}{R_2} = \frac{1}{R_o} \left(V' + kA_V V' \right) + \frac{1}{R_2} \left(V' - kV' \right).$$

Op-amp buffer: output resistance (continued)



Non–inverting amplifier



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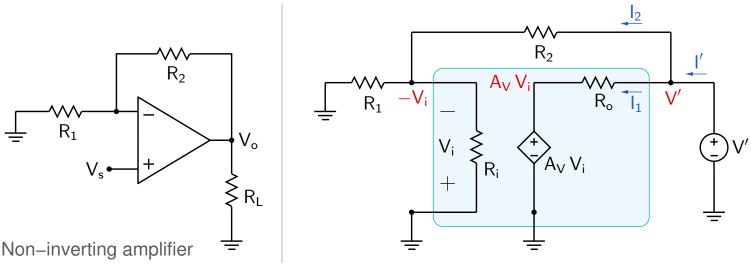
$$I' = I_1 + I_2 = \frac{V' - A_V V_i}{R_o} + \frac{V' - (-V_i)}{R_2} = \frac{1}{R_o}(V' + kA_V V') + \frac{1}{R_2}(V' - kV').$$

$$\frac{I'}{V'} = \frac{1}{R_o}(1 + kA_V) + \frac{1}{R_2}(1 - k) \rightarrow R_{out} = \frac{V'}{I'} = \frac{R_o}{(1 + kA_V)} \parallel \frac{R_2}{(1 - k)} \approx \frac{R_o}{(1 + kA_V)}$$



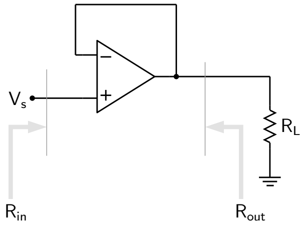
$$V_i = -\frac{(R_i \parallel R_1)}{R_2 + (R_i \parallel R_1)} V' \equiv -kV'.$$

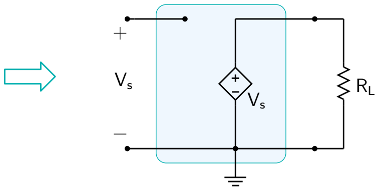
$$I' = I_1 + I_2 = \frac{V' - A_V V_i}{R_o} + \frac{V' - (-V_i)}{R_2} = \frac{1}{R_o}(V' + kA_V V') + \frac{1}{R_2}(V' - kV').$$

$$\frac{I'}{V'} = \frac{1}{R_o}(1 + kA_V) + \frac{1}{R_2}(1-k) \rightarrow R_{out} = \frac{V'}{I'} = \frac{R_o}{(1+kA_V)} \parallel \frac{R_2}{(1-k)} \approx \frac{R_o}{(1+kA_V)}$$

Special case: Op-amp buffer

$$k = \frac{(R_i \parallel R_1)}{R_2 + (R_i \parallel R_1)} \rightarrow 1 \quad \Rightarrow \quad \boxed{R_{out} \approx \frac{R_o}{1 + A_V}}$$





In summary, the buffer (voltage follower) provides

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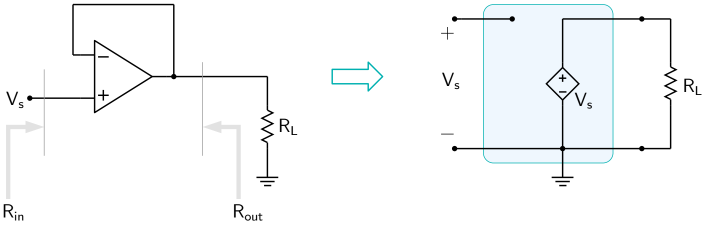

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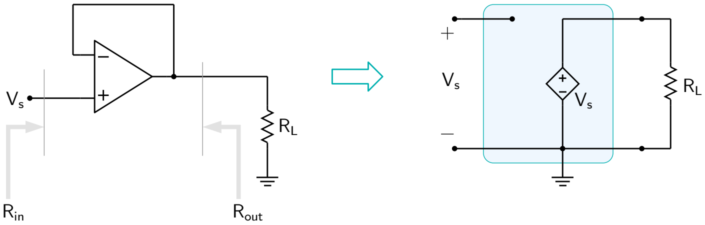

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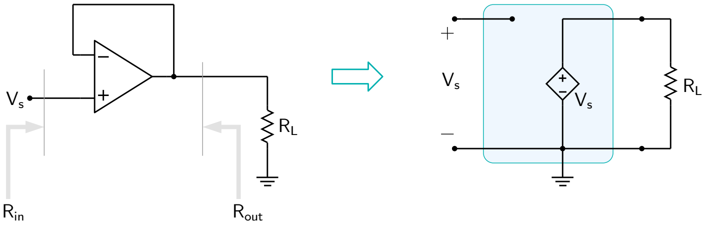
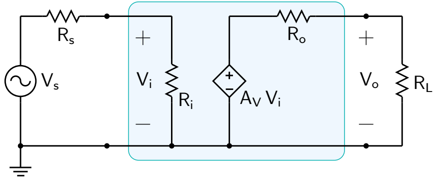


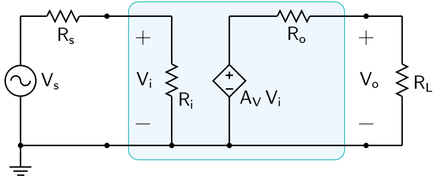
In summary, the buffer (voltage follower) provides

* a large input resistance R_{in} as seen from the source.
* a small output resistance R_{out} as seen from the load.
* a gain of 1, i.e., the output voltage simply follows the input voltage.

Loading effects (revisited)



Problem: We would like to have $V_o = A_V\ V_s$.

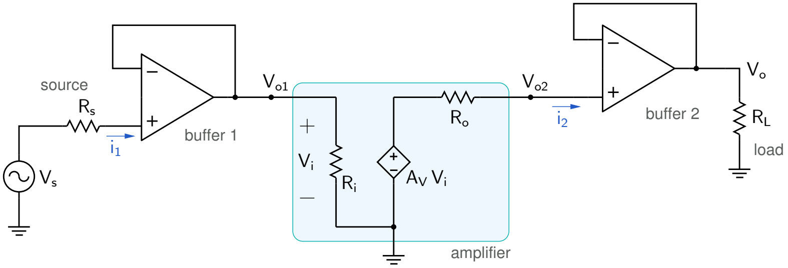


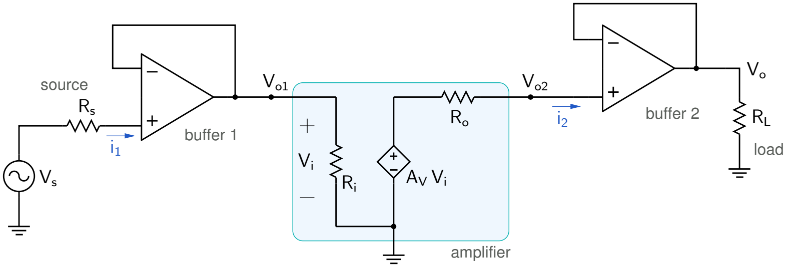
Problem: We would like to have $V_o = A_V\, V_s$.

But the actual output voltage is,

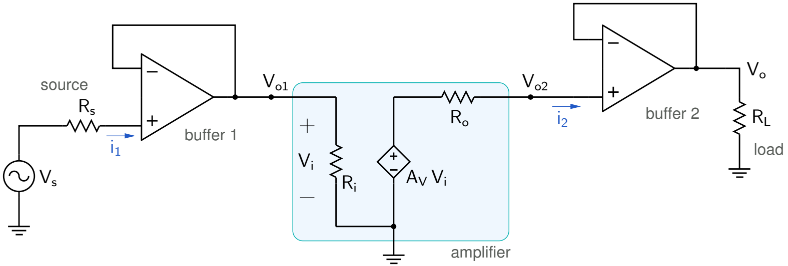
$$V_o = \frac{R_L}{R_o + R_L}\, A_V\, V_i = A_V\, \frac{R_L}{R_o + R_L}\, \frac{R_i}{R_i + R_s}\, V_s .$$

Op-amp buffer





Since the buffer has a large input resistance, $i_1 \approx 0\,A$,
and V_+ (on the source side) $= V_s \rightarrow V_{o1} = V_s$.



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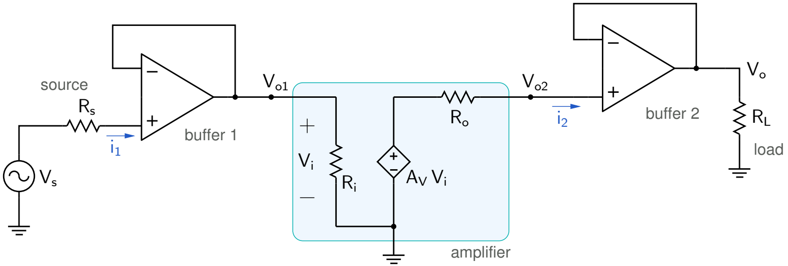

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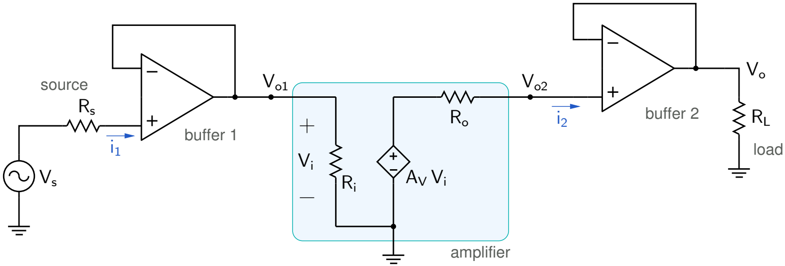

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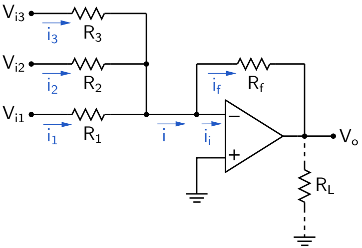
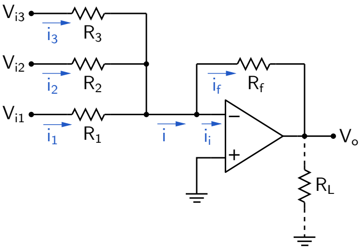
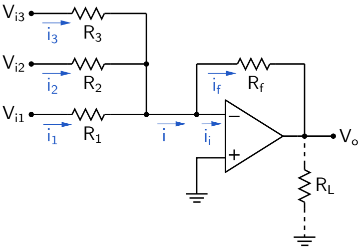
Note that the load current is supplied by the second buffer which acts as a voltage source ($= A_V\,V_s$) with zero source resistance.

Op-amp circuits (linear region)





$V_{-} \approx V_{+} = 0\ V \rightarrow i_1 = V_{i1}/R_1,\ i_2 = V_{i2}/R_2,\ i_3 = V_{i3}/R_3\ .$



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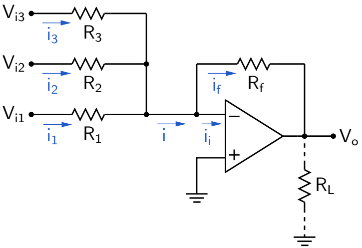

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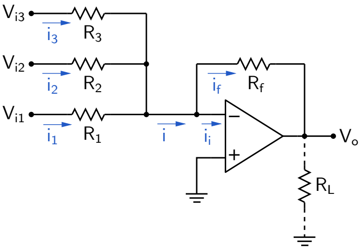

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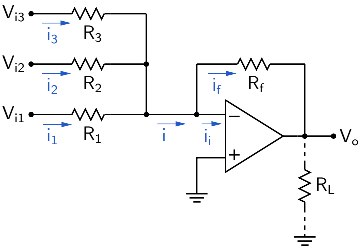
$$V_o = V_- - i_f\,R_f = 0 - \left(\frac{V_{i1}}{R_1} + \frac{V_{i2}}{R_2} + \frac{V_{i3}}{R_3}\right) R_f = -\left(\frac{R_f}{R_1}\,V_{i1} + \frac{R_f}{R_2}\,V_{i2} + \frac{R_f}{R_3}\,V_{i3}\right),$$

i.e., V_o is a *weighted sum* of V_{i1}, V_{i2}, V_{i3}.



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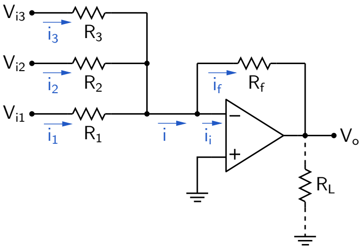
If $R_1 = R_2 = R_3 = R$, the circuit acts as a summer, giving

$\boxed{V_o = -K\,(V_{i1} + V_{i2} + V_{i3})}$ with $K = R_f/R\,.$

Summer example



$R_1 = R_2 = R_3 = 1\ k\Omega$

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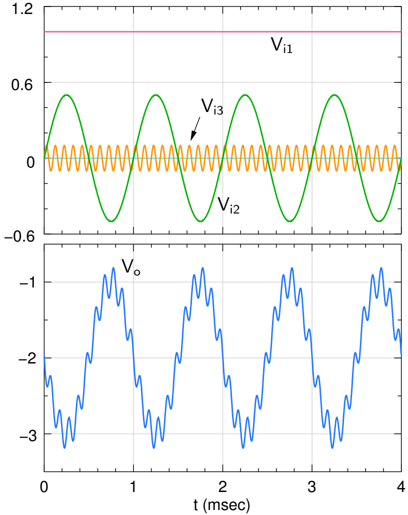
SEQUEL file: `ee101_summer.sqproj`



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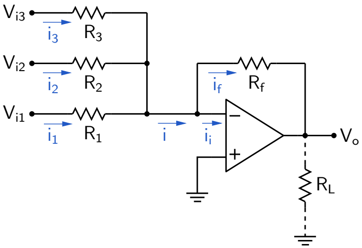

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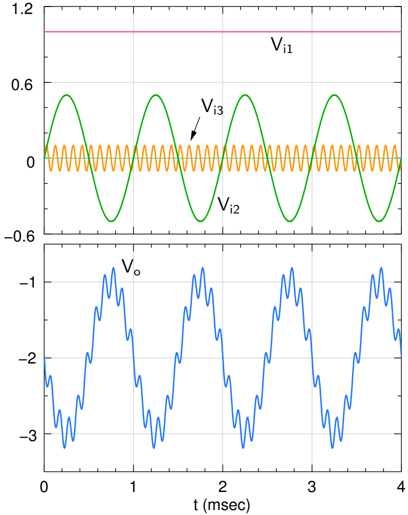

∗ Note that the summer also works with DC inputs (so do inverting and non-inverting amplifiers).

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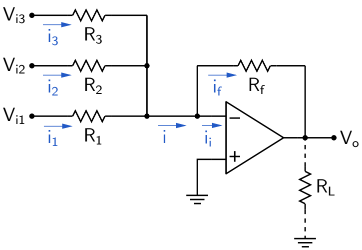

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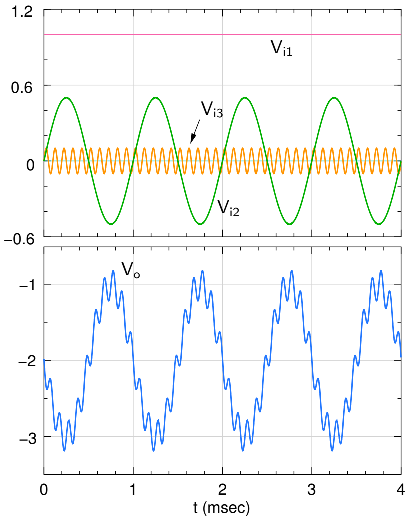

* Note that the summer also works with DC inputs (so do inverting and non-inverting amplifiers).
* Op-amps make life simpler! Think of adding voltages in any other way.

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* Typical resistance values: 0.1 k to 100 k.

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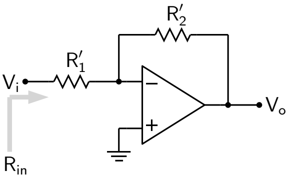

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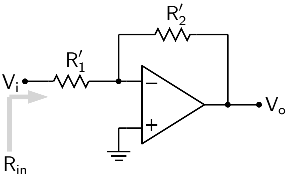

$$R_{\text{in}} = R'_1 = 10\,\text{k}.$$

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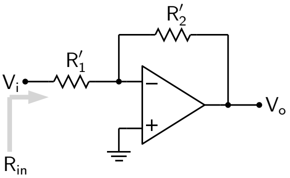

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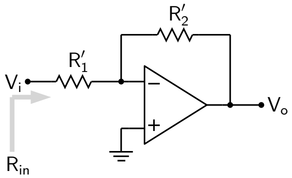

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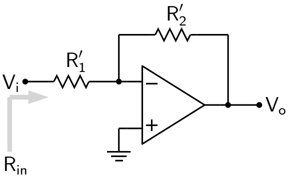

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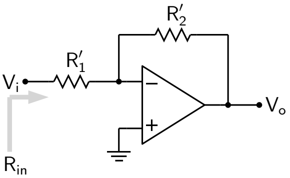

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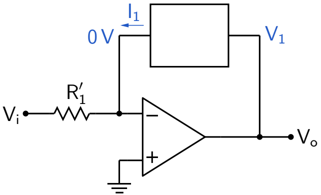

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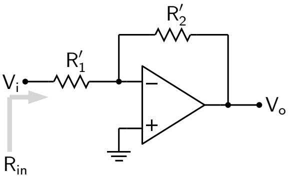

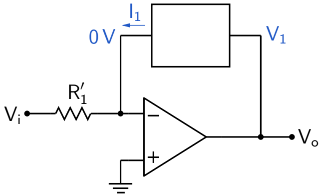

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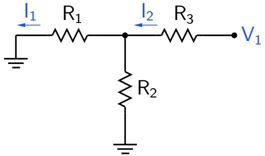
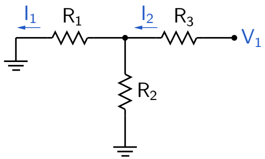
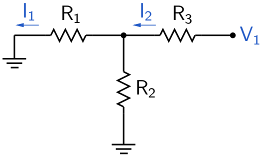
If we ensure $\dfrac{V_1}{I_1} = R_2'$, we will satisfy the gain condition.


I1
R1
I2
R3
V1
R2



$$I_2 = \frac{V_1}{R_3 + (R_1 \parallel R_2)}$$



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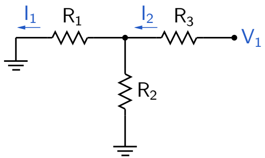

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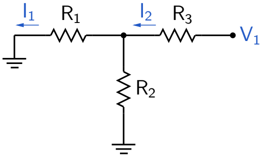

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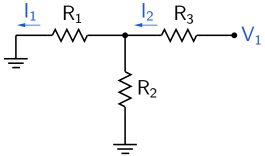
$\rightarrow$ Choose $R_1,\ R_2,\ R_3$ such that $R_{\text{eff}} = R_2' = 1\,\text{M}\Omega$.



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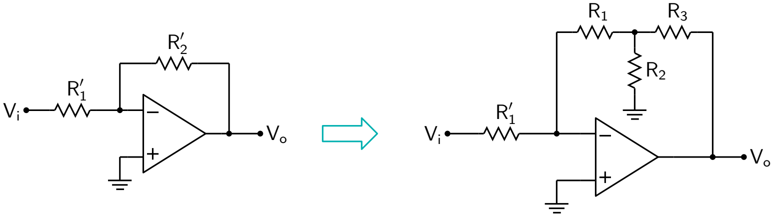
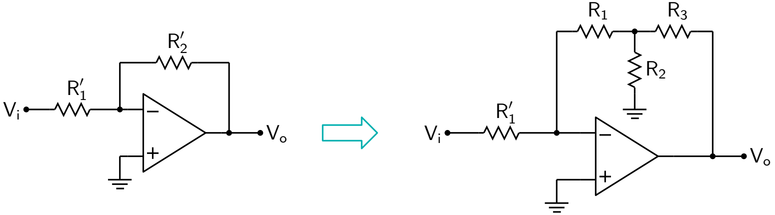
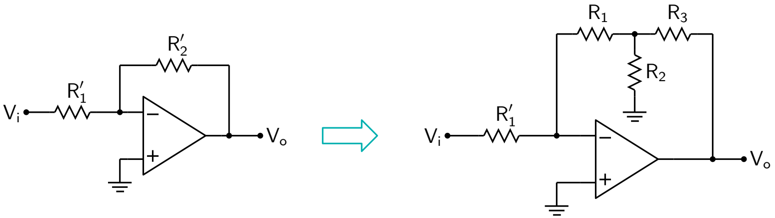
→ Choose R_1, R_2, R_3 such that $R_{\text{eff}} = R_2' = 1\,\text{M}\Omega$.





$$R_{\text{eff}} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

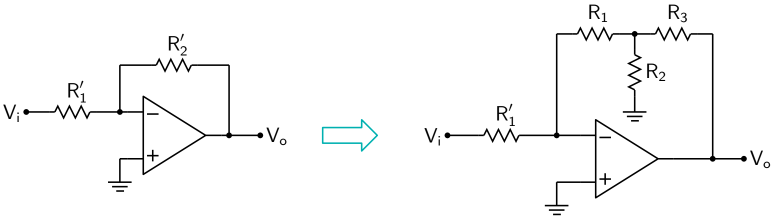
We want $R_{\text{eff}} = R_2' = 1\,\text{M}\Omega$.



$$R_{\text{eff}} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

We want $R_{\text{eff}} = R_2' = 1\,\text{M}\Omega$.

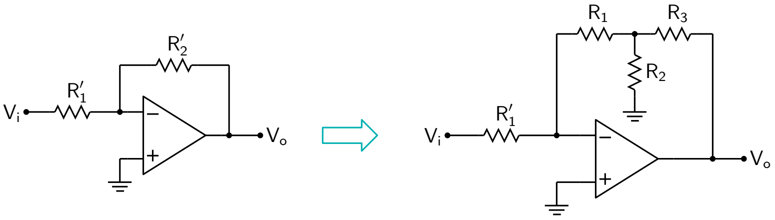
Let $R_1 = R_3 \equiv R$



$$R_{\text{eff}} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

We want $R_{\text{eff}} = R_2' = 1\,\text{M}\Omega$.

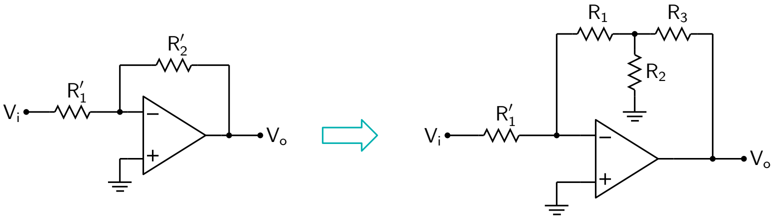
Let $R_1 = R_3 \equiv R \rightarrow R_{\text{eff}} = \dfrac{R^2 + 2\,R\,R_2}{R_2} = R\left(\dfrac{R}{R_2} + 2\right)$



$$R_{\text{eff}} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

We want $R_{\text{eff}} = R_2' = 1\,\text{M}\Omega$.

Let $R_1 = R_3 \equiv R \rightarrow R_{\text{eff}} = \dfrac{R^2 + 2\,R\,R_2}{R_2} = R\left(\dfrac{R}{R_2} + 2\right) \rightarrow R_2 = \dfrac{R}{\dfrac{R_{\text{eff}}}{R} - 2}$

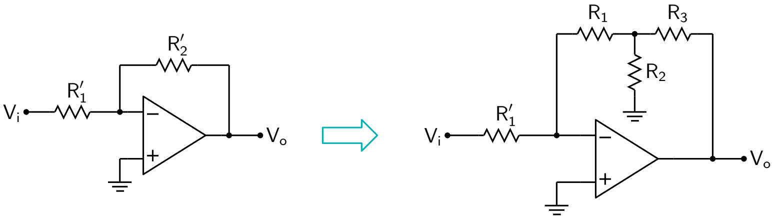


$$R_{\text{eff}} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

We want $R_{\text{eff}} = R_2' = 1\,\text{M}\Omega$.

Let $R_1 = R_3 \equiv R \rightarrow R_{\text{eff}} = \dfrac{R^2 + 2\,R\,R_2}{R_2} = R\left(\dfrac{R}{R_2} + 2\right) \rightarrow R_2 = \dfrac{R}{\dfrac{R_{\text{eff}}}{R} - 2}$

For $R = 10\,\text{k}$, $R_2 = \dfrac{10\,\text{k}}{100 - 2} \approx 102\,\Omega$.



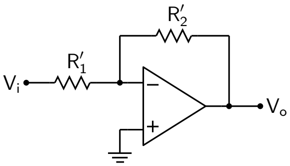
$$R_{\rm eff} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

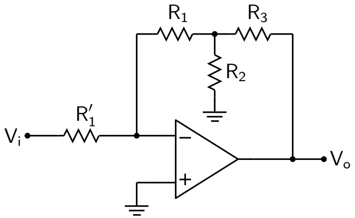
We want $R_{\rm eff} = R_2' = 1\,{\rm M}\Omega$.

Let $R_1 = R_3 \equiv R \rightarrow R_{\rm eff} = \dfrac{R^2 + 2\,R\,R_2}{R_2} = R\left(\dfrac{R}{R_2} + 2\right) \rightarrow R_2 = \dfrac{R}{\dfrac{R_{\rm eff}}{R} - 2}$

For $R = 10\,{\rm k}$, $R_2 = \dfrac{10\,{\rm k}}{100 - 2} \approx 102\,\Omega$.

Ref: Wait et al, *Introduction to op-amp theory and applications*, McGraw-Hill, 1992.



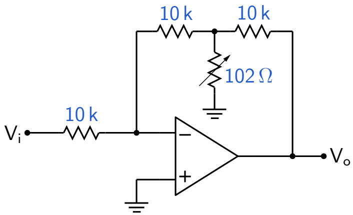


$$R_{\text{eff}} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

We want $R_{\text{eff}} = R_2' = 1\,\text{M}\Omega$.

Let $R_1 = R_3 \equiv R \rightarrow R_{\text{eff}} = \dfrac{R^2 + 2\,R\,R_2}{R_2} = R\left(\dfrac{R}{R_2} + 2\right) \rightarrow R_2 = \dfrac{R}{\dfrac{R_{\text{eff}}}{R} - 2}$

For $R = 10\,\text{k}$, $R_2 = \dfrac{10\,\text{k}}{100 - 2} \approx 102\,\Omega$.



Ref: Wait et al, *Introduction to op-amp theory and applications*, McGraw-Hill, 1992.